# Investigation of Stability of Sway Precast RC Frames with Semi-Rigid Beam to Column Connections

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*Abstract:* Prefabricated concrete systems provide complicated structural elements which support speedy construction of engineering facilities, but with difficulties in designing their beam-to-column connections as well as proper account of the semi-rigid stability analysis of structural frames where such connections exist. In this study, a simple beam-to-column U-bar connection is proposed and analyzed along with standard patent joints commonly used for precast framed buildings. A relationship between the joint moment capacity and rotation was developed, on the basis of which the lateral stability analysis of a high-rise prefabricated concrete building frame containing the proposed joint was performed. The model was analytically derived from stress-strain theory and elastic linear analysis. The developed model when incorporated into frame analysis accounts for both non-linear semi-rigid behavior in connections and P- $\Delta$  effects of connecting beam and column members. It gives choice to the designer, to change member cross sections and connection parameters interactively with ease, and yields favorable results when compared with other models. Result of lateral stability analysis of the model frame was verified with proven computer software program. Designers therefore have a new form of connector for use in construction of precast concrete buildings.

## 1. INTRODUCTION

A new form of semi-rigid connection is declared along with a structural model to account for its non-linear semi-rigid behavior in frames. Equally developed is a non-linear elastic analytical technique for the frame analysis. The method accounts for both connection non-linearity and geometric non-linearity in frames. It does not rely on the use of charts or iterative solutions. Moment-resisting connections are mostly found in foundations and beam to column joints. To design a semi-rigid joint as a safe moment-resisting connection capable of withstanding sagging moments in the beam or increasing global frame stiffness, a moment of resistance of at least 75-100kN.m is required amd if less the connection is better designed as pinned joint (Elliot, 2002), (Elliot et al, 2003, 2004), (Ferreira & Elliot 2002), (Waddell, 1974). But, in concrete with insitu monolithic connection, slipping or bond failure at joints can be resisted by anchoring bars in form of U bents or hooks to fully develop the design stress (Mosley et al, 1999). Thus, it seems that designing precast concrete connection as shown in Figure 1, may enhance flexural stiffness and improve anchorage. Comparison between the proposed U-bar connector and other commercial connectors was conducted in this research. This type of beam-column connector has not been used in any previous studies. A major economic factor of the U-connector is the relative ease of material availability, because all its component elements are locally fabricated from readily available steel rebars and plates.

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Figure 1: Composite action of the Proposed U-bar connector

In this research, a typical two storey precast concrete frame with four types of semi-rigid connections (billet, single cleat, welded and proposed U-bar connector) as well as monolithic rigid joint, were analyzed to show applicability of the proposed frame elastic non-linear analysis and also to arrive at important conclusions regarding effect of the connections on sway buckling and lateral deflection in multistory unbraced frames. The basic characteristics of semi-rigid connections are defined by the connector rotation, connector ultimate moment and calculated moment capacity. Hence to incorporate the non-linear semi-rigid behavour of connections into frame analysis, moment-rotation characteristics obtained through experimental models (Elliot et al., 2003) was used, which are favored to others such as linear model, Polynomial model, B-spline model, Power models and Finite element. Elliot et al., (2003), carried out full scale laboratory test on patented precast billet, cleat and welded connections to obtain moment-rotation characteristics as shown in Table1 and also derived analytical equations to predict their behavior. The frame behavior was studied under combined horizontal and gravity loads. Results of analysis were validated using STAAD pro computer software. Also the effect of beam-to-column stiffness ratio ( $K_B/K_C$ ) on behavior of frame with U-connector was studied in order observe the effect of connection rigidity on the sideway stability of semi-rigid precast concrete frames.

Type of	Connection mode of failures	<b>Connector rotation</b>	Connector test	Calculated
connection		θ (rad)	ultimate moment	moment capacity
			(KN.m)	M <sub>RC</sub> (KN.m)
Billet connector (single side beam size 600mm)	Failed in tension (hogging) by slipping of the top of locating cleat, inducing cracks and crack widening	0.00666	-58.1	-106.1
Single cleat connector (single side, beam size 600mm)	Failed in both sagging and hogging mode at the bolted connection between beam and cleat. Very poor in strength.	0.00256	-13.2	-17.4
Welded connection (single side, beam size 600mm	Failed by bending ad shear at the bottom of beam interface.	0.00820	-154.3	-153.1

Table 1: Moment-Rotation	<b>Characteristics for</b>	Semi-Rigid	Connections
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Source: Elliot et.al the Structural Engineer, 2003

## 2. RESEARCH METHODOLOGY

#### 2.1 The Proposed Connector

The connector is a precast-to-precast interface type. It consists mainly of 2 nos. diameter 32mm projected U-seating bars, a high friction bolt, U-anchor bar and an angle-bar. The U-seating bar is partly inserted into column and partly projects out of the column face. L-shape steel plate (100mm x 100mm x 12mm thick), with two slot opening, is inserted into the beam, using the U-anchor. Part of the beam is void to allow for upward passing of threaded end U-seating bar. The connection completes by bolting the top of angle-plate with the bolt into the high friction grid nut inserted in the column. The threaded end of U-bar at top of the plate is equally bolted. The annulus of the joint is then filled with Grade 30N/mm<sup>2</sup> concrete mix.

#### 2.2 Structural Mechanism of Proposed Connector

The U-bar provides anchorage and resists shear. The high friction nut when fully tightened provides resistance to slippage and tension. The angle plate transfers moment from the beam to column. The concrete grout around the joint resists compression. Adopting rectangular stress block approach according to BS8110 and using the usual notation for rebars and sections, the hogging moment of resistance of the connector can be calculated as follows from Figure 2.



Figure 2: Equilibrium of Forces in the U-connector Component

(1)

$$M_{RC} = (F_{bolt} + F_{Anchor plate} + F_{U-dowel bar}) Z$$

$$Z = (d_c - 0.45X_c)$$

$$X_C = \frac{F_{bolt} + F_{panchor plate} + F_{U-bar}}{0.67 f_{cu} \ 0.9b}$$
(2)

where, d<sub>c</sub> is the depth to centriod of the summation of all internal forces.

Z, is effective moment arm of friction bolt from top of column

 $f_{cu}$  is cube strength of concrete

#### 2.3 Calculation of the Rotation of Connector

Developing the moment-rotation relationship of the proposed connection it was assumed that the total relative end rotation of the joint  $\theta$  is the sum of the rotational deformation of individual components of which the joint is formed

Hence, 
$$\theta_{\text{joint}} = \left[ \frac{f_b t_L}{8E_b h_b^2} + \frac{d_L^2 f_L h_L}{E_L t_L^3} + \frac{f_U h_U}{E_U d_U^2} \right]$$
 (3)

Where

E <sub>b</sub>	=	Elastic modulus of steel bar
$\mathbf{h}_{\mathbf{b}}$	=	Moment arm of bolt
$d_b$	=	Diameter of bolt
$h_L$	=	Effective moment arm of plate
$b_{\rm L}$	=	Breath of plate
$L_L$	=	Length of plate
$t_{\rm L}$	=	Thickness of plate
$\mathbf{h}_{\mathrm{UI}}$	=	Effective height of short length of seating U-bar above L-plate acting as a dowel
$h_{U2}$	=	Effective height of long length of seating U-bar below L-plate acting as dowel
$E_U$	=	Elastic modulus of U-bar
$d_{\mathrm{U}}$	=	Diameter of U-bar
$h_A$	=	Effective height of U-anchor above the beam
$E_A$	=	Elastic modulus of U-anchor bar
$\mathbf{f}_{\mathrm{L}}$	=	Yield stress of Grate 43 steel plate

## 2.4 Rotational Stiffness of the Proposed Connection

The rotational stiffness S, of the beam-to-column joint is expressed as;

 $S = M_{RC}/\theta_{connector}$ 

#### 2.5 Estimating Semi-rigidity of the Proposed Connector

#### The rigidity of the connection can be estimated (Degertekin, 2004; Monforton, 1963) as:

$$K_{\rm K} = 1/1 + 3EI/K_{\rm J}L$$

Where,  $K_J$  is the connector rotational stiffness accounting for semi-rigidity,  $K_K$  is the rigidity factor expressed in terms of  $K_J$  and end rotation of a framing member. For  $K_K = 0$  joint is pinned,  $K_K = 1$  joint is rigid and anything in-between is semi-rigid behavior.

$$K_{\rm J} = M_{\rm RC} / \theta_{\rm J} \tag{6}$$

Where  $\theta_J$  = connector rotation,  $M_{RC}$  = moment of resistance of the joint.

## 3. ANALYSIS OF CRITICAL SWAY LOAD IN THE PROPOSEDPRECAST FRAME WITH THE U-BAR CONNECTOR

Assume that the portal frame in Figure 3, has rigid joints at the ends (B & C) and firmly fixed at the base, and the length of beam and column are different. Then there exist three degree sof freedom at the joints ( $\theta_B$ ,  $\theta_C$  and  $\Delta$ ) resulting from gravity and sway loading. Linear elastic stiffness matrix equation of the frame is given in form of equation 7.

(4)

(5)



Figure 3: Deformed Model for Sway Critical Load in Semi-Rigidly Connected Frame

$$\begin{vmatrix}
M_{AB} \\
M_{BA} \\
M_{BA} \\
M_{BC} \\
M_{CB} \\
M_{CD} \\
M_{DC}
\end{vmatrix} = K_{AB} \begin{vmatrix}
4 & 2 \\
2 & 4 & 1 \\
4 & 2 \\
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2 & 1 & 1 & 1 \\
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where  $n = K_{beam(BC)} / K_{column(AB)}$  is the ratio of beam stiffness to column stiffness, and  $J = K_{col(AB)} / K_{col(CD)}$  is the ratio of column stiffness to column stiffness in the frame i.e  $K_{AB} = K_{BC} = K_{CD} = EI/L$ , chord rotations =  $\Delta/L$ ,  $M_{AB}$  = moment at A in span AB,  $M_{BA}$  = moment at B in span AB,  $M_{BC}$  = moment at B in span BC,  $M_{CB}$  = moment at C in span BC,  $M_{CD}$  = moment at C in span CD and  $M_{DC}$  = moment at D in span CD.

For moment equilibrium at joint B:

$$\Sigma M_{\rm B} = M_{\rm BA} + M_{\rm BC} \tag{8}$$

$$\Sigma M_{\rm C} = M_{\rm BC} + M_{\rm CD} \tag{9}$$

For shear equilibrium of force

$$[M_{BA} + M_{AB}]/L + [M_{CD} + M_{DC}]/L$$
(10)

The relationship between the end moment and end-rotation of a beam with rigid joints are expressed as:

$$M_{\rm B} = EI/L \begin{bmatrix} 4\theta_B & 2\theta_c \end{bmatrix}$$
(11)  
$$M_{\rm C} = EI/L \begin{bmatrix} 2\theta_C & 4\theta_B \end{bmatrix}$$
(12)

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Suppose joints B and C are replaced with the proposed semi-rigid connection. Then equations 11 and 12 can be written by replacing the end rotations  $\theta_B$  and  $\theta_C$  in the equation with semi-rigid ends rotations  $\theta_B - \theta_b$  and  $\theta_C - \theta_c$ , where  $\theta_b$  and  $\theta_c$  are relative rotations.

Thus the equations of beam with the semi-rigid connector becomes

$$M_{\rm B} = EI/L \left[ 4 \left( \theta_{B} - M_{B} / K_{JB} \right) + 2 \left( \theta_{C} - M_{C} / K_{JC} \right) \right]$$
(13)

$$M_{C} = EI/L[4(\theta_{C} - M_{C}/K_{JC}) + 2(\theta_{C} - M_{B}/K_{JB})]$$
(14)

where,  $K_{JB}$  and  $K_{JC}$  are rotational stiffness of the connector expressed as:

$$K_{JB} = M_B / \theta_b \text{ and } K_{JC} = M_C / \theta_c \tag{15}$$

Expressing the equation in terms of  $\theta_B$  and  $\theta_C$  and  $K_K$ 

$$\mathbf{M}_{\mathrm{B}} = EI/L \big[ r_{BB} \,\theta_B \,+\, r_{BC} \,\theta_C \,\big] \tag{16}$$

$$M_{\rm C} = EI/L[r_{CB}\,\theta_B + r_{CC}\,\theta_C] \tag{17}$$

where,  $r_{BB}=12K_{KB}$  /4 -  $K_{KB}\;\;K_{KC}$  and  $r_{BC}=r_{BB}/2$ 

Assuming the same rotational stiffness at both ends,  $K_{KB} = K_{KC} = K_K$ 

For simplicity let  $r_1 = r_{BB} = r_{CC}$  and  $r_2 = r_{BC} = r_{CB}$ , then

$$r_1 = 12K_K / 4 - K_K^{2}$$
(18)

$$r_2 = r_1/2$$

The rigidity of the column ends is similarly expressed in the form of equation 18, given as follows

$$J_1 = 12K_K/4 - K_K^2 \text{ and } J_2 = J_1/2$$
  
Where,  $J_1 = J_{AA} = J_{BB} = J_{CC} = J_{DD}$  and  $J_2 = J_{BA} = J_{AB} = J_{DC} = J_{CD}$ 

It is important to note that the symbol J was adopted to clarify different contributions of beam and column members meeting at joints B and C. Also n and m are  $K_{BC}/K_{AB}$  and  $K_{CD}/K_{AB}$  respectively.

Therefore, the stiffness matrix equation of the frame becomes;



Suppose an axial load *P* and a sway load H act on the frame, the frame deflects as shown in Figure 3. At the critical condition, the vertical load *P* reaches its critical value  $P_{cr}$  and the applied horizontal load H produces a deflection  $\Delta_{max}$  prior to formation of mechanism at the feet. Thus, the frame sway critical load is given in the form in equation 21

$$\mathbf{P}_{\rm cr} = (\mathrm{H}/\Delta_{\rm max}) \,\mathbf{h} \tag{21}$$

Where,  $\Delta$  = sway deflection; h = height of storey. Geometric effects due to direct axial compression in the columns and shear deformation are not considered. The overall frame stiffness in equation 20 is a 6 x 6 matrix transformed from local axis to global axis considering three degrees of freedom (DOF)  $\theta_B$ ,  $\theta_C$  and  $\Delta$  given below as:

(19)

It is important to note that  $J_1$  and  $J_2$  are rigidity factors for joints A and D. From principle of contragradience,  $K_S = T^T K^T$ . Transpose matrix  $T^T$  is

$$T^{T} = \begin{vmatrix} -1/h & -1/h & 0 & 0 & -1/h & -1/h \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{vmatrix}$$
(23)

$$T^{T}K = \begin{vmatrix} J_{2} & J_{1} & n\frac{h}{L}r_{1} & n\frac{h}{L}r_{2} & 0 & 0 \\ 0 & 0 & n\frac{h}{L}r_{2} & n\frac{h}{L}r_{1} & mJ_{1} & mJ_{2} \\ \frac{-(J_{2}+J_{1})}{h} & \frac{-(J_{2}+J_{1})}{h} & 0 & 0 & \frac{-m(J_{2}-J_{1})}{h} & \frac{-m(J_{2}-J_{1})}{h} \end{vmatrix}$$
(24)

Therefore, frame stiffness matrix K<sub>S</sub> is given in the form

$$\begin{vmatrix} M_{B} \\ M_{C} \\ H \end{vmatrix} = K_{AB} \begin{vmatrix} \frac{nr_{1}h + J_{1}}{L} & \frac{nr_{2}h}{L} & \frac{-(J_{2} + J_{1})}{h} \\ \frac{nr_{2}h}{L} & \frac{nr_{1}h + mJ_{1}}{L} & \frac{-m(J_{2} + J_{1})}{h} \\ \frac{-(J_{2} + J_{1})}{h} & \frac{-m(J_{2} + J_{1})}{h} \\ \frac{-(J_{2} + J_{1})}{h} & \frac{-m(J_{2} + J_{1})}{h} \\ \frac{M_{B}}{H} \end{vmatrix}$$
(24)

At the critical load

 $M_B=0,\ M_C=0$ 

 $P_{cr} = K_{S}h \text{ or } (H/\Delta_{max})h$ 

where  $K_s$  is the lateral stiffness at beam level given as  $H/\Delta_{max}$ , H is a horizontal load. The axial stiffness of columns AB and CD is given as

$$K_{AB} = \frac{EI}{h} \tag{26}$$

From equation 24,  $\Delta = \frac{H}{\varphi K_{AB}}$  where  $\varphi$  is an expression obtained from the matrix in terms of  $\Delta$  only i.e  $\theta_{B}$  and  $\theta_{C}$  are

expressed in terms of  $\Delta$ .

Substituting  $\Delta$  and  $K_{AB}$  in equation 21 and assuming the two columns share  $P_{cr}$  equally.

$$P_{cr} = \phi K_{AB} = 0.5 \phi EI \tag{27}$$

Recall that critical condition of the frame is prior to formation of mechanism at the feet, thus Euler load for a pined ended strut applies.

That is  $P_E = EI\pi^2/h^2$ , substituting this equation 27 gives

(25)

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$$P_{cr} = \left[0.5 \ \varphi * \frac{h^2}{\pi^2}\right] P_E \tag{28}$$

where,  $K_{AB}$  =axial column stiffness,  $h_{AB}$  = height of column, n is  $K_{beam}/K_{column}$  ratio and  $J_1$  and  $J_2$  are rigidity factors as in equation 18 and 19.

#### 3.1 Sway Deflection of Semi-Rigid Frame

The sway deflection of frame is predicted from conventional cantilever deflection of a member given below assuming  $P_{cr}$  is known.

 $P_{cr} = Kh \text{ or } (3EI/h^2)\Delta$ 

$$\Delta = P_{cr}h/3EI$$

(29)

#### **3.2 A Numerical Example**

The RC frame selected for analysis is shown in Figure 4. The problem is to determine the sway critical load of the frame with the assumption of semi-rigid joint connections. Given that  $E = 26 \times 10^3 \text{ N/mm}^2$ 



Figure 4(a): Typical Three-Storey Frame Selected for Analysis

Using the proposed method, frame is reduced to three sub-frames below:

In the figure below,  $H_1$ ,  $H_2$  and  $H_3$  are horizontal loads.



Figure 4(b): Diagram of the Typical Three-Storey Frames under Static Load

The individual frame critical load is determined from equation 28, where

$$P_{cr} = \left[ \varphi \quad \frac{h^2}{4\pi^2} \right] P_E$$

$$K_{beam} = EI_b / L_b = 6.99 \ X \ 10^{10} ; \quad K_{col(lower)} = EI_{col} / L_{col} = 1.33 \ X \ 10^{10}$$

$$K_{col(upper)} = EI_{col} / L_{col} = 1.55 \ X \ 10^{10} ; \quad J = Kcol_{AB} / Kcol_{CD} = 1.00$$

$$n = \frac{n_s K_{beam}}{2}$$

Where,  $n_s = no$  of stories considered from bottom,  $K_b$  and  $K_c$  stiffness of beam and column, n = ration of beam stiffness to column stiffness.

For ground storey;

 $n_{K_{column}}$ 

$$n_1 = \frac{\left(K_{B1} + K_{B2} + K_{B3}\right)}{n(K_{C1} + K_{C2})} = 4.98$$

For 2<sup>nd</sup> storey;

$$n_2 = \frac{\left(K_{B2} + K_{B3}\right)}{\left(n \times K_{C2}\right)} = 4.50$$

For 3<sup>rd</sup> storey

$$n_3 = \frac{K_{B3}}{K_{C3}} = 5.25$$

Overall  $P_{creq}$  for the multistory was calculated with reference to Orumu (1997) in Eqn. 30, where for multi-storey frames with loads  $W_1, W_2, W_3$  and  $W_n$  acting on each floor from top to bottom as shown in Figure 3, the equivalent critical load ( $P_{creq}$ ) was determined as the load weighted average of the various  $P_{cr}$ . Thus,

$$P_{creq} = \frac{W_1 P_{cr1} + (W_1 + W_2) P_{cr2} + (W_1 + W_2 + W_3) P_{cr3}}{3W_1 + 2W_2 + W_3}$$
(30)

Euler load of (lowest column) is obtained as  $P_E = EI \pi^2 / L^2 = 4886.3 \text{ kN}$ 

In summary, the analytical procedure can be summarized as:

- a) Calculate the moment of resistance, rotation and rotational stiffness of connections according to the momentrotation model.
- b) Compute rigidity factor  $K_K$  of the connection
- c) Plot sway-bucking load versus K<sub>K</sub>
- d) Plot sway deflection versus K<sub>K</sub> of semi-rigid connections ensuring limit of sway drift does not exceed H<sub>storey</sub>/400

#### 3.3 Validation of results

STAAD Pro computer software was used to validate the result of this analysis. The software is chosen because of its versatility. It is one of the powerful packages for building frame analysis. It can be used for static analysis, modal analysis, harmonic analysis, buckling analysis and P-delta analysis. The computational process is carried out on the typical frame under consideration.

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## 4. ANALYSIS AND DISCUSSION OF RESULTS

Detailed results are given in the Tables and plots in the subsections.

#### 4.1 Computation of Rigidity factor and Stiffness factors

These factors are computed as per Equation 18 and displayed in Table 2, for four semi-rigid connections used in the frame analysis.

Type of connector	Connector Rotation (rad)	Calculated θ <sub>J</sub> Moment Resistance M <sub>RC</sub> (kNm)	of	$\begin{array}{c} Cord  Rotational \\ Stiffness \\ K_J \!=\! M_{RC}\!/\theta_J \\ (kN.m/rad) \end{array}$	rı	r <sub>2</sub>	K <sub>K</sub> Rigidity Factor
Billet	0.00660	-106.1		16060.60	1.15	0.57	0.371
Cleat	0.00256	-17.4		3810.78	0.60	0.30	0.199
Welded	0.00820	-153.1		18670.73	1.27	0.63	0.407
U-bar	0.00598	-142.0		23913.86	1.48	0.74	0.467
Monolithic	0.0000	-350		3.500E+10	4.00	2.00	1.00

#### Table 2: Comparison of Connection Parameters for Frame Analysis

Table 3: Comparison of Estimated Sway Parameters with other Connections

Types of Connection	Rigidity Factor $K_K$	Sway-buckling Load P <sub>cr</sub> (kN)	Sway (Δmm)	Deflection	Maximum h/400	Sway
Billet	0.37	2788	0.00307			
Cleat	0.10	1730	0.00200			
Welded	0.41	2729	0.00331		0.04875	
U-bar	0.47	2950	0.00368			
Monolithic	1.00	3491	0.00381			

## 4.2 Result of Analysis of Frame with Varying $K_B/K_C$ and $P_{cr}/P_E$ Ratios

In the frame under consideration, moment of inertia  $I_{col}$  of the column member was varied, while that of the beam remained constant which led to various values  $K_B/K_C$  ratios generated for the U-bar connector joint component of the frame. From results shown in Table 2 above, it can be seen that moment capacity of the U-bar connector ( $M_{RC} = 142kNm$ ) is up to the requirement for moment-resisting connections, and the rigidity factor ( $K_K = 0.47$ ) of the connector compares favorably to other connectors. It was found in Table 4 that an increase in column size when connection rigidity in the beam is constant causes an increase in sway resistance of the frame. The U-bar may behave like a monolithic connector when  $K_B/K_C$  ratio equals 0.15. It was observed that the minimum value of axial load causing critical condition in semi-rigid frames increases as the connector rigidity increases. The sway deflection of semi-rigid frame with U-bar connection under sway is within allowable limit of H/400.

			U-bar		Mono		U-bar	Mono
Column size (mm)	$I_{col} (mm^4)$	K <sub>B</sub> /K <sub>C</sub> ratio	P <sub>cr</sub>	$P_{E}$	P <sub>cr</sub>	P <sub>E</sub>	$P_{cr}/P_{E}$	
230*230	20788188571	5.24	2950	4886.3	3491	4886.3	0.603	0.714
300*300	60171428571	1.81	4208	14144.4	4243	14144.4	0.297	0.299
350*350	1.11475E+11	0.97	5980	26202.4	6195	26202.4	0.228	0.236
400*400	1.90171E+11	0.57	8627	44700.1	8967	44700.1	0.192	0.206
450*450	3.04618E+11	0.37	8906	71600.9	9065	71600.9	0.124	0.126

			U-bar		Mono		U-bar	Mono
Column size (mm)	$I_{col} (mm^4)$	K <sub>B</sub> /K <sub>C</sub> ratio	P <sub>cr</sub>	$P_{\rm E}$	P <sub>cr</sub>	$P_{\rm E}$	$P_{cr}/P_{E}$	
230*230	20788188571	5.24	2777	4886.3	2820	4886.3	0.568	0.577
300*300	60171428571	1.81	4135	14144.4	4280	14144.4	0.297	0.302
350*350	1.11475E+11	0.97	6050	26202.4	6115	26202.4	0.230	0.233
400*400	1.90171E+11	0.57	8810	44700.1	8975	4470.1	0.197	0.199
450*450	3.04618E+11	0.37	8910	71600.9	9070	71600.9	0.124	0.126

Table 5: Load for U-Connector and Monolithic Joint Obtained From STAAD Pro Model



Figure 5: Plot of P<sub>cr</sub>/P<sub>E</sub> versus K<sub>B</sub>/K<sub>C</sub> for U-bar and Monolithic Connector

## 4.3 Validation of Results with STAAD pro. Software

Results of the sway-buckling load and sway deflection obtained by the proposed method were compared to results obtained using STAAD pro software. The beam end stiffness release in the STAAD pro model was taken as the rotational stiffness of the connector expressed in kN.m/degree. The minimum critical sway load in computer model was determined by gradually increasing applied load P until buckling took place. The critical load for monolithic frame predicted as  $0.76P_E$  according to (Home and Merchant, 1987) was also compared with result obtained by the proposed method for monolithic frame. The results from the STAAD pro model compares favorably to the proposed model as seen in Table 6, which validates the proposed model as safe.

Types	Sway-buckling load (P <sub>cr</sub> )kN			Sway deflection (Δ)m			
	PROP	STAAD	%DIFF	PROP	STAAD	%DIFF	
Billet	2788	2852	+2.23	0.00308	0.00440	-20.90	
Cleat	1730	2349	+26.35	0.00190	0.00210	-9.52	
Welded	2729	2911	+6.25	0.00332	0.00450	-26.22	
U-bar	2950	2977	+0.91	0.00357	0.00470	-24.04	
Monolithic	3491	2833	-23.22	0.00389	0.00500	-28.53	

Table 6+	Comparison	of Result	of STAAD	model and	Pronosed	Model
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## 5. CONCLUSION

The proposed model when incorporated into frame analysis gives values that compare favorably with that obtained with standard computer software. Commercial connectors at exterior, interior and corner joints of prefabricated concrete high-rise building subjected to hogging moments and opposing sway moments have been studied and additional data generated for further study.

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The main conclusion is that connections which incorporate continuity in the bars or combine composite slab action provides appreciable rigidity and resist side sway deflection better the non-complaint ones. Connector with rigidity factor less than 0.2 should be regarded as pinned.

From the foregoing, the following design considerations are recommended:

- 1) The  $M_{RC}$  of the connector is based on an internal couple between concrete at the end of the beam in compression and tensile components of the connector. BS 8110 stress block approach is used.
- For typical precast concrete beam and column sizes (300-600mm deep), M<sub>RC</sub> should be at least 75kNm for use in semi-rigid design.
- 3) The axial force capacity of the tie steel, or the tensile components if no tie steel exist, should be at least 0.07.
- Connection flexibility/geometric non-linearity of framing members should be fairly treated in semi-rigid design to minimize errors.
- 5) Experimental validation of the proposed U-bar connector is recommended to fully explore its practical use.

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